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**Programs of Invited Lectures
and
Abstracts of Research Reports**

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Programs of Invited Lectures

Locally-global principle in the fields theory and its application.

Vasyl' I. Andriychuk

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1. The Brauer group of a field.
2. Locally global principle (theorems Minkovski–Gasse types) and its applications to the arithmetics of algebraic groups.

Basilica group

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The Basilica group is generated by an automat with tree states over two-letter alphabet. It also can be defined as the group of iterated monodromy of the polynomial $z^2 - 1$.

This group has very interesting properties and provides a solution to an important problem. We shall survey properties of this group and methods of its investigation as well as will demonstrate its application to analysis,

dynamical systems and random walks. Finally we shall pose some related open problems.

Compact semigroups

I. I. Guran

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In the lecture we discuss on the structure of compact topological semigroups.

Topological inverse semigroups

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1. Inverse semigroups. Topologizing of inverse semigroups.
 2. (0-)Simple inverse semigroups. Embedding of topological inverse semigroups into simple topological inverse semigroups with different algebraic and topological properties. (0-)Simple pseudo-compact (countably compact, compact) topological inverse semigroups. Congruence-free pseudo-compact topological inverse semigroups.
 3. Continuity of inverse in locally compact and countably compact inverse topological semigroups.
 4. Free topological inverse semigroups.
 5. The topology of a compact topological inverse Clifford semigroup (Yeager Theorem).
-

Model theory

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1. Structures and theories (Languages and structures. Theories. Definable sets and interpretability).
2. Basic techniques (The compactness theorem. Complete theories. Vaught's test, completeness of algebraically closed fields, Ax's theorem. Up and down elementary embeddings, Lowenheim-Skolem, back and forth).
3. Algebraic examples (Quantifier elimination, algebraically closed fields, real closed fields, Hilbert's 17th problem, cell decomposition).

Functional representations of dual symmetric Fock spaces associated with compact groups

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In lectures is planned to present properties of symmetric Fock spaces, associated with some weakly compact groups of unitary operators given in Hilbert spaces. Final result of these researches is the fact that hermitian dual spaces to such Fock spaces have structure of Hardy classes of analytic functions in some infinite dimensional Banach invariant domains.

The lectures include the following subjects

- 1 Hilbert-Schmidt Polynomials
- 2 Weakly Compact Groups of Unitary Operators
- 3 Uniformly Algebras of Weakly Continuous Analytic Functions
- 4 Infinite Products of Invariant Measures
- 5 Formulas of Decomposition for Infinite Products of Invariant Measure
- 6 Functional Representations of Symmetric Fock Spaces
- 7 Hardy Spaces on Invariant Banach Domains

In submitted lectures the following literature is used [1–10].

References

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- [2] S. Dineen, *Complex Analysis on Infinite Dimensional Spaces*, Monographs in Mathematics, Springer, New York, 1999.
- [3] T. W. Gamelin, *Analytic Functions on Banach spaces*, in Complex Potential Theory, NATO Adv. Sci. Inst. Ser. C. Math. Phys. Sci., vol. 439, Kluwer Acad. Publ., 1994.
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- [9] R. A. Ryan, *Application of topological tensor products to infinite dimensional holomorphy*, Ph.D. Thesis, Trinity College Dublin, 1980.
- [10] M. Valdivia, R. M. Aron, and C. Hervés, *Weakly continuous mappings on Banach spaces*, J. Funct. Anal. **52**.

Morse functions on surfaces

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Let M be a smooth compact surface, $f : M \rightarrow \mathbb{R}$ be a smooth (C^∞) function. Recall that a point $z \in M$ is *critical* for f if in some local coordinates (x, y) near z we have $f'_x(z) = f'_y(z) = 0$.

A critical point z of f is *non-degenerated* if the Hessian of f at z

$$H(f, z) = \begin{pmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{pmatrix}$$

is a non-degenerated matrix. In this case there are (possibly another) local coordinates (u, v) at z in which f is a quadratic form:

$$f(u, v) = f(z) \pm u^2 \pm v^2.$$

Notice, that, if z is *not* a critical point for f , then there are local coordinates at z in which $f(x, y) = x$.

A smooth function $f: M \rightarrow \mathbb{R}$ is *Morse*, if all critical points of f are nondegenerated and lay in the interior of M .

Let $\mathcal{M}(M, \mathbb{R})$ be the subset of $C^\infty(M, \mathbb{R})$ consisting of Morse functions. This set is open and everywhere dense in $C^\infty(M, \mathbb{R})$. Thus almost every smooth function on M is Morse.

Similarly we can define a Morse function from M into the circle S^1 and consider the space $\mathcal{M}(M, S^1) \subset C^\infty(M, S^1)$.

There is a natural action of the group of diffeomorphisms $\text{Diff}(M)$ of M on the space $C^\infty(M, \mathbb{R})$ defined by the formula $h \cdot f = f \circ h^{-1}$, where $f \in \mathcal{M}(M, \mathbb{R})$ and $h \in \text{Diff}(M)$. Then for every $f \in C^\infty(M, \mathbb{R})$ the orbit O_f and the stabilizer S_f of this function are well-defined.

The aim of these lectures is to explain some structural properties of $\mathcal{M}(M, \mathbb{R})$ and $\mathcal{M}(M, S^1)$. We will consider the following two questions

1. Description of the connected components of the spaces $\mathcal{M}(M, \mathbb{R})$ and $\mathcal{M}(M, S^1)$.
2. Homotopy types of the orbits of Morse functions on M under the above action of $\text{Diff}(M)$.

The basic idea of Morse theory is that the structure of a generic smooth function on a manifold determines many homotopical properties of this manifold. On surfaces this theory is in some sense complete: indexes of critical points of a Morse function on surface completely determine topological type of this surface. Thus surfaces give simple examples of manifolds on which key aspects of Morse theory can easily be viewed.

In these lectures we will concentrate on some homotopical properties of the space of all Morse functions on a compact surface. We will emphasize very close connection between this space and the group of diffeomorphisms.

Lecture 1. *Surfaces and Morse functions.* Definition on surfaces, types of surfaces, orientable and non-orientable surfaces and relationships between them. Real- and circle-valued Morse functions. Their structure and relation to the topology of surface. Classification of compact surfaces via Morse functions.

Lecture 2. *Diffeomorphisms of surfaces.* We will study the diffeotopy group of surfaces. This is the group of connected components of the group of all diffeomorphisms. Two main types of diffeomorphisms will be defined. These are so called Dehn twists and Y -diffeomorphisms. We will give the proof of the fact that they generate diffeotopy group.

Lecture 3. *Connected components of the space of Morse functions.* In this lecture we will give a proof of the classification of connected components of the space of Morse functions using generators of diffeotopy group of surfaces. We will emphasize that there is a representation (by A.Hatcher and W.Thurston) of homeotopy group via Morse functions. Thus there is a very close relation between diffeomorphisms and Morse functions.

Lectures 4. *Homotopy type of orbits of Morse functions.* We will consider the action of the group of diffeomorphisms of surface on the space of all smooth functions. The aim of these two lectures is to calculate the homotopy type of the connected components of orbits of Morse functions under the action above. To do this we will recall first the classification of homotopy types of the group of diffeomorphisms of surfaces. Then we will calculate the homotopy types of the stabilizers of Morse functions. This will allow us to obtain the information about the homotopy type of orbits.

Automatic continuity of group homomorphisms

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Topological dynamics and combinatorics

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We begin with famous Poincare Recurrence Theorem. At least two modern mathematical theories grew up from this theorem: topological dynamics and ergodic theory. We intend to prove two fundamental theorems from topological dynamics and to show the interplay of these theorems with combinatorics of numbers.

Furstenberg-Weiss Theorem. *Let X be a compact metric space, T_1, \dots, T_m be pairwise commuting homeomorphisms of X . Then there exists a point $x \in X$ and an increasing sequence $(k_n)_{n \in \omega}$ of natural numbers such that*

$$T_1^{k_n}(x) \rightarrow x, \dots, T_m^{k_n}(x) \rightarrow x, n \rightarrow \infty$$

Following Furstenberg, we extract from this statement the van der Waerden Theorem: for every finite coloring of \mathbb{N} there exists an arbitrary long monochrome arithmetic progression. We sketch also the ergodic proof of Szemerédi Theorem: every subset of \mathbb{N} of positive upper density has an arbitrary long arithmetic progression.

Auslander-Ellis Theorem. *Let X be a compact metric space, $T: X \rightarrow X$ be a continuous mapping, $x \in X$. Then there exists a uniformly recurrent point $y \in X$ such that x and y are proximal.*

A point y is called uniformly recurrent in the dynamical system (X, T) if, for every $\varepsilon > 0$, there exists a natural number m such that, for every natural number n , at least one of the points $T^n(y), \dots, T^{n+m}(y)$ belongs to the ε -ball around y . Two points x, y are called proximal if, for every $\varepsilon > 0$, there exists $n \in \mathbb{N}$ such that $d(T^n(x), T^n(y)) < \varepsilon$, where d is the metric on X .

To prove this theorem we use so-called enveloping (or Ellis) semigroup of dynamical system. From this theorem we extract the **Hindman Theorem**: *for every finite partition of $\mathbb{N} = A_1 \cup \dots \cup A_m$, there exists a cell A_i of the partition and an infinite subset $A \subseteq A_i$ such that $FS(A) \subseteq A_i$, where $FS(A)$ is the set of all finite sums of distinct elements of A .*

Our exposition follows "Lectures-essay in Combinatorics" by I. V. Protasov (in preparation).

History of Lviv mathematics

Ya. G. Prytula

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In the lecture we discuss on the history of Lviv mathematics.

$L - 2$ cohomology and applications

V. Sharko

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1. Smooth manifolds, riemannian metric, differential forms, covariant differentiation.
 2. De-Rham complex, Hodge theorems, integration on manifolds.
 3. Homology groups, fundamental group, universal cover, group ring.
 4. C^* -algebra and Hilbert modules connected with manifolds.
 5. $L - 2$ theory, Atiah conjecture.
-

The classification problem of locally finite simple groups

W. I. Suszczansky

Kyiv Taras Shevchenko National University, Kyiv, Ukrainea

1. The classification of simple groups. Characterizing basic series of finite simple groups.
2. Locally finite countable simple groups. Kegel coverings. Simple linear groups over locally finite fields. Finitary linear groups. Finitary altering group.

3. Results of Ulrich and Mayerfrankfeld. Non-finitary simple locally finite groups. Partitions on types. Diagonal embeddings and groups of diagonal type. Classification of diagonal limits of direct sums of finite altering groups.
 4. Homogeneous symmetric and altering groups. Bratelli diagrams and induced transformation groups. Criterion of simplicity. Examples. Action on trees.
-

Algebraic topology

Mykhailo M. Zarichnyi

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1. Operations on topological spaces. Cell complexes. Homotopies. Homotopy extension theorem. Approximation by cell maps.
 2. Homotopy groups. Relations between absolute and relative homotopy groups. Homotopy exact sequences (pairs, fiber bundles).
 3. Singular homology. Homology of *CW*-complexes. Relation between homology and homotopy. Hurewicz theorem. Cohomology.
 4. Vietoris homology. Homology of coarse spaces.
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Abstracts of Research Reports

A universality theorem for compact topological Clifford inverse semigroups

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A famous Ponizovsky Decomposition theorem asserts that each commutative inverse semigroup S embeds into the product $\prod_{e \in E} H_e^0$ of its Ponizovsky factors. In this talk we shall discuss topological counterparts of this Ponizovsky Decomposition theorem.

Recall that a set S equipped with an associative operation $*$: $S \times S \rightarrow S$ is called *an inverse semigroup* if for every element $x \in S$ there exists a unique element of S denoted by x^{-1} and called *the inverse element* of x , such that $x * x^{-1} * x = x$ and $x^{-1} * x * x^{-1} = x^{-1}$. If for every element x of inverse semigroup S holds $x * x^{-1} = x^{-1} * x$ then S is called *Clifford inverse semigroup*.

Each Clifford inverse semigroup S decomposes into the sum $S = \bigcup_{e \in E} H_e$ of maximal subgroups $H_e = \{x \in S \mid x * x^{-1} = x^{-1} * x = e\}$ corresponding to idempotents e of S . The set $E = \{e \in S \mid e * e = e\}$ of all idempotents of S is called the maximal semilattice of S . If a Clifford inverse semigroup S is given with a topology such that the maps $*$: $S \times S \rightarrow S$ and $*^{-1}$: $S \rightarrow S$ are continuous, then S is called *a topological Clifford inverse semigroup*. A topological semilattice admitting a base of the topology, consisting of subsemilattices is called a *Lawson semilattice*.

The analogue of the Ponizovsky factors for topological Clifford inverse semigroups are conical semigroups. By a *conical semigroup* we understand a semigroup S of the form $S = G \times E/G \times I$, where G is a topological group, E is a topological semilattice and I is closed ideal in E . For example, $\text{Cone}(G) = G \times [0, 1]/G \times \{0\}$, $G^0 = G \times \{0, 1\}/G \times \{0\} = G \cup \{0\}$ are conical semigroups. Consider the unit circle $\mathbb{T} = \{z \in \mathbb{C} \mid |z| = 1\}$ as topological group, with respect to multiplication of complex numbers, then $\mathbb{T}^0 = \{z \in \mathbb{C} \mid |z| \in \{0, 1\}\}$ is the centered circle, considered as a commutative inverse semigroup, with respect to multiplication of complex numbers and $\text{Cone}(\mathbb{T}) = \mathbb{D} = \{z \in \mathbb{C} \mid |z| \leq 1\}$ is the unit disk considered as a commutative inverse semigroup with respect to the following operation $(r_1, \varphi_1) * (r_2, \varphi_2) = (\max(r_1, r_2), \varphi_1 + \varphi_2)$.

Theorem 1. *Each compact topological Clifford inverse semigroup S with Lawson maximal semilattice E is a subsemigroup of the product $S \subset \prod_{e \in E} \text{Cone}(H_e)$.*

Theorem 2. *Each compact topological Clifford inverse semigroup S with Lawson zero-dimensional maximal semilattice E is a subsemigroup of the product $S \subset \prod_{e \in E} H_e^0$.*

Theorem 3. *Each compact topological Clifford inverse semigroup S with Lawson maximal semilattice E is a subsemigroup of the product $S \subset (\prod_{n \in \mathbb{N}} \text{Cone}(O(n)))^\tau$, where $O(n)$ is the orthogonal subgroup and $\tau = w(S)$ is the weight of S .*

Theorem 4. *Each compact topological Clifford inverse semigroup S with Lawson zero-dimensional maximal semilattice E is a subsemigroup of the product $S \subset (\prod_{n \in \mathbb{N}} (O(n)^0))^\tau$, where $O(n)$ is the orthogonal subgroup and $\tau = w(S)$ is the weight of S .*

In abelian case the theorem implies an interesting universality result:

Corollary 1. *Each commutative compact topological inverse semigroup S with Lawson maximal semilattice E is a subsemigroup of the power \mathbb{D}^τ of the disk, where $\tau = w(S)$ is the weight of S .*

Corollary 2. *Each commutative compact topological inverse semigroup S with zero-dimensional semilattice E is a subsemigroup of the power $(\mathbb{T}^0)^\tau$ of the centered circle, where $\tau = w(S)$ is the weight of S .*

References

- [1] P. A. Grillet, *Semigroups: An Introduction to the Structure Theory*, Marsel Dekker, 1995.

Coars topologies on function spaces

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The fine topology (limitation topology, in another terminology) on function spaces has obtained various applications in infinite-dimensional topology and differential topology. There exists a counterpart of this topology, which we call the coars topology, on the set of maps from a coars space (i.e. a space endowed with a coarse structure in the sense of J. Roe) into a metric space. Some fundamental facts concerning this topology are established. We also formulate some problems concerning connectedness, Baire property and normality of the obtained function spaces.

The Borg-type theorem for generalized Jacobi matrices

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The tridiagonal block matrix

$$H = H_{[0,N]} := \begin{pmatrix} A_0 & \tilde{B}_0 & & \mathbf{0} \\ B_0 & \ddots & \ddots & \\ & \ddots & \ddots & \tilde{B}_{N-1} \\ \mathbf{0} & & B_{N-1} & A_N \end{pmatrix}, \quad (1)$$

where A_j are the companion matrices for real monic polynomials p_j , $k_{j+1} \times k_j$ matrices B_j and $k_j \times k_{j+1}$ matrices \tilde{B}_j are given by

$$B_j = \begin{pmatrix} 0 & \dots & b_j \\ \dots & \dots & \dots \\ 0 & \dots & 0 \end{pmatrix}, \quad \tilde{B}_j = \begin{pmatrix} 0 & \dots & \varepsilon_j \varepsilon_{j+1} b_j \\ \dots & \dots & \dots \\ 0 & \dots & 0 \end{pmatrix} \quad (b_j > 0, \varepsilon_j = \pm 1),$$

will be called a *generalized Jacobi matrices* associated with the sequences of polynomials $\{\varepsilon_j p_j\}_{j=0}^N$ and numbers $\{b_j\}_{j=0}^{N-1}$. The matrices of the form (1)

arise in the Pade approximant theory and the indefinite moment problem (see [1]). The matrix H defines a cyclic symmetric operator in the space with indefinite inner product $\langle \cdot, \cdot \rangle_G$. The trace formula for generalized Jacobi matrices

$$\operatorname{tr} A_0^l = \operatorname{tr} H^l - \operatorname{tr} H_{[1, N]}^l \quad (1 \leq l < k_0 + k_1)$$

is obtained. The analog of Borg theorem for the one rank perturbation of matrix H having the form

$$H_\tau = H - \tau \langle \cdot, e \rangle_G e, \quad e = (\delta_{0i})_{i=0}^n.$$

is proved. Namely, *two spectra $\sigma(H)$ and $\sigma(H_\tau)$ uniquely determine the generalized Jacobi matrix H and τ* . The proof is based on the trace formula and boundary operator method and Weyl function [2].

References

- [1] M. Derevyagin, and V. Derkach, *Spectral problems for generalized Jacobi matrices*, Linear Algebra Appl. **382** (2004), 1–24.
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On modified probability measure functor in the coarse category of proper metric spaces

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Let \mathcal{C} be the category whose objects are proper metric spaces and whose maps are proper large scale uniform maps (see, e.g. [1] for the definition).

Given a proper metric space, (X, d) , we define the space $\mathcal{P}(X)$ by letting $\mathcal{P}(X) = \{(\mu, A) \in P(X) \times \exp X \mid \operatorname{supp} \mu \subset A\}$. Here $P(X)$ is the space of probability measures on X with compact supports endowed with the Kantorovich-Rubinshtein metric and $\exp X$ is the hyperspace of X endowed with the Hausdorff metric, both induced by the metric d ; the space $\mathcal{P}(X)$ is equipped the metric induced by the l_1 -metric on the product.

It is proved that \mathcal{P} is a functor on the coarse category and that there exist natural transformations $\eta: 1_{\mathcal{C}} \rightarrow \mathcal{P}$, $\psi: \mathcal{P}^2 \rightarrow \mathcal{P}$ such that the triple (\mathcal{P}, η, μ) is a monad on the category \mathcal{C} (see [2]).

One can define the tensor product operation $\otimes: \mathcal{P}(X) \times \mathcal{P}(Y) \rightarrow \mathcal{P}(X \times Y)$, which turns out to be a coarse map, i.e. a morphism in \mathcal{C} .

We also investigate the properties of openness and bicommutativity for the functor \mathcal{P} .

References

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Chiral phases

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We study the construction of the affine Coxeter groups. It has been shown how to construct the decomposition on the semidirect product of the investigated groups Coxeter diagram. Having used this decomposition we give the list of irreducible representations of affine Coxeter groups and constructed the enveloping C^* -algebra of a group $*$ -algebra of affine Coxeter groups.

If $S = \{s_1, s_2, \dots, s_n\}$ is the set of generators and $m_{s_i, s_k}: S \times S \rightarrow \mathbb{N} \cup \{\infty\}$ such that $m_{s_i, s_i} = 1, i = 1..n, m_{s_i, s_j} > 1$, for all distinct i and j , then $W = G\langle s_1, s_2, \dots, s_n \mid (s_i s_j)^{m_{s_i, s_j}} = e \rangle$ is called a *Coxeter group*. It is convenient to associate a Coxeter group with the graph called a *Coxeter diagram*. Its nodes are indexed by S . If m_{s_i, s_j} is 3 or more then there exists an edge between nodes s_i and s_j . If $m_{s_i, s_j} > 3$ we connect s_i and s_j by edge labelled with m_{s_i, s_j} .

Some propositions, that was proofed.

Theorem 1. *The decomposition of the affine Coxeter group in semidirect product $\mathbb{Z}^{n-1} \rtimes G_{fin}$ can be obtained using path-tracing method for corresponding Coxeter graph, that consists of special passing around of the graphs.*

Theorem 1. *Let W be an affine group and $W \simeq \mathbb{Z}^{n-1} \rtimes G_{fin}$. Then, enveloping C^* -algebra of the group $*$ -algebra of W is C^* -algebra \mathcal{A} of matrix-functions, $f(\varphi_1, \varphi_2, \dots, \varphi_{n-1})$, $|G_{fin}| \times |G_{fin}|$ continuous on $0 \leq |\varphi_1| \leq |\varphi_2| \leq \dots \leq |\varphi_{n-2}| \leq \varphi_{n-1} \leq \pi$. If the following properties are fulfilled for the non-empty set of indexes $\{i_1, i_2, \dots, i_t\}$*

$$\begin{aligned} & \varphi_{i_1} = \varphi_{i_2} = \dots = \varphi_{i_t} = 0, \varphi_k \neq 0 \text{ and } \varphi_k \neq \varphi_j \\ & \quad \forall k \neq j : k, j \notin \{i_1, i_2, \dots, i_t\} \\ \text{or } & \varphi_{i_1} = \varphi_{i_2} = \dots = \varphi_{i_t} = \varphi_k, k \notin \{i_1, i_2, \dots, i_t\}, \end{aligned}$$

than the functions $f(\varphi_1, \varphi_2, \dots, \varphi_{n-1})$ are a block-diagonal matrix-functions with equivalent blocks, that correspond to the class $\mathcal{K}(c_1, \dots, c_{n-1})$ with parameters $c_i = e^{i\varphi_i}$.

Around Grasshopper Lemma

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Let G be a finite connected graph with the set of vertices $V(G)$. Given any $u, v \in V(G)$, we denote by $d(u, v)$ the length of the shortest path between u and v . By Grasshopper Lemma, there exists a numeration v_1, \dots, v_n of $V(G)$ such that

$$d(v_1, v_2) \leq 3, \dots, d(v_{n-1}, v_n) \leq 3, d(v_n, v_1) \leq 3.$$

This numeration is called the *grasshopper circle in G* . We propose an algorithm of linear complexity for construction of grasshopper circle in arbitrary graph.

A graph G is called *quasihamiltonian* if there exists a numeration v_1, \dots, v_n of $V(G)$ such that

$$d(v_1, v_2) \leq 2, \dots, d(v_{n-1}, v_n) \leq 2, d(v_n, v_1) \leq 2.$$

This numeration is called the *quasihamiltonian circle*. It is still unknown whether the test-problem "Quasihamiltonian graph" is NP-complete as well as there are no characterizations for the class of quasihamiltonian graphs.

Among couple of sufficient conditions the most remarkable is the **Fleischer Theorem**: *every 2-connected graph is quasihamiltonian*. We give the following necessary conditions.

Let G be a quasihamiltonian graph, $v \in V(G)$. After deletion of v , G disintegrates into connected components G_1, \dots, G_k . Then at most two of them that contain more than two vertices are connected with v by one edge.

We show that the above conditions are sufficient for graphs in which any two minimal circles have at most one common edge. Then we apply this statement to show that the following graphs are quasihamiltonian: line graph, round-about reconstruction of graph, interval graphs. Moreover, for these cases we give an algorithm for construction of quasihamiltonian circle.

We call a graph G to be hamiltonian connected if, for any two vertices u, v , there exists a hamiltonian circle in G in which u and v are adjacent. If G is hamiltonian connected then for any three vertices x, y, z there exist at least two paths from x to y avoiding z . We do not know if this condition is sufficient for graph to be hamiltonian connected.

On Sylow's structure of homogeneous symmetric group of superdegree p^∞

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Let p be any prime, V_n be n -dimensional vector space over the residue field Z_p . Every permutation u from symmetric group $S(V_n)$ corresponds the collection $\langle f_1(x_1, \dots, x_n), f_2(x_1, \dots, x_n), \dots, f_n(x_1, \dots, x_n) \rangle$ of reduced polynomials over Z_p . Collections

$$\langle x_1 + a_1, x_2 + a_2(x_1), \dots, x_n + a_n(x_1, \dots, x_{n-1}) \rangle,$$

such that $a_i(x_1, \dots, x_{i-1})$ are reduced polynomials ($i = 1, 2, \dots, n$), forms the Sylow p -subgroup P_n of the group $S(V_n)$ [1].

Define strictly diagonal [2] embeddings $\delta_n^{(i)}$ of the symmetric group $S(V_n)$ into the symmetric group $S(V_{n+1})$ ($i = 0, 1, \dots, n; n = 1, 2, \dots$) as follows: $\delta_n^{(i)}(\langle f_1, \dots, f_n \rangle) = \langle g_1, g_2, \dots, g_{n+1} \rangle$, where

$$g_k(x_1, \dots, x_{n+1}) = \begin{cases} x_{i+1}, & \text{if } k = i + 1, \\ f_k(x_1, \dots, x_i, x_{i+2}, \dots, x_{n+1}), & \text{if } k \neq i + 1. \end{cases}$$

Let $S_{p^\infty} = \lim\langle S(V_n), \delta_n^{(i_n)} \rangle$. If $\overline{S(V_n)}$ is the image of $S(V_n)$ for natural embedding $S(V_n) \rightarrow S_{p^\infty}$ then $S_{p^\infty} = \bigcup_{n=1}^{\infty} \overline{S(V_n)}$.

Every sequence of strictly diagonal embeddings $\delta_n^{(i_n)}$ ($0 \leq i_n \leq n$, $n \in N$) and some Sylow p -subgroup of $P_m < S_{p^m}$ ($m \geq 1$) determines the Sylow p -subgroup of S_{p^∞} : $P(m, \vec{i}) = \bigcup_{j=m}^{\infty} \overline{P_j^{z_j}}$, where $\vec{i} = (i_1, i_2, \dots)$, $\overline{P_n}$ is the image of P_n in $\overline{S(V_n)}$, $\overline{P_j^{z_j}} > \overline{P_{j-1}^{z_{j-1}}}$ for $j = m+1, m+2, \dots$ and permutations $z_j \in S(V_j)$ is assigned by the sequence of embeddings: $z_m = \varepsilon$, $z_j = w_j^{(i_{j-1})} \cdot z_{j-1}$, $w_j^{(i)} = \langle x_1, x_2, \dots, x_{i+1}, \dots, x_j, x_i \rangle$ ($j = m+1, m+2, \dots$).

Let $\Delta = \{\langle i_1, i_2, \dots \rangle \mid 0 \leq i_n \leq n, n \in N\}$ and $(m)\vec{i} = (i_m, i_{m+1}, \dots)$, where $\vec{i} = (i_1, i_2, \dots) \in \Delta$.

Theorem. 1) Subgroups $P(m, \vec{i})$, $P(n, \vec{j})$, $m \leq n$, $\vec{i} = \langle i_1, i_2, \dots \rangle \in \Delta$, $\vec{j} = \langle j_1, j_2, \dots \rangle \in \Delta$ are coincide if and only if $i_m = m, \dots, i_{n-1} = n-1$ and $(n)\vec{i} = (n)\vec{j}$.

2) Subgroups $P(m, \vec{i})$, $P(n, \vec{j})$, $m \leq n$, are conjugate if and only if $(s)\vec{i} = (s)\vec{j}$ for some $s \in N$.

So, the set $\{P(1, \vec{i}) \mid \vec{i} \in \Delta\}$ of Sylow p -subgroups is continual and contains continuum classes of conjugate subgroups.

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On estimates for minimal and maximal differential operators

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About unbounded perturbations of the mixed non-homogeneous parabolic boundary problems

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Let in the bounded domain $\Omega \subset \mathbb{R}^n$ with boundary of the class C^∞ the strongly elliptic operator $L(x, D) := \sum_{|\alpha| \leq 2m} a_\alpha(x) D^\alpha$, ($a_\alpha(x) \in L_\infty(\Omega)$) is determined. We assume, that $a_\alpha(x)$ under the condition $|\alpha| = 2m$ are continuous in $\overline{\Omega}$. Further $a(x, \xi) := \sum_{|\alpha|=2m} a_\alpha(x) \xi^\alpha$, $\xi^\alpha := \xi^{\alpha_1} \cdot \dots \cdot \xi^{\alpha_n}$. Let on $\partial\Omega$ is given normal system of the boundary operators $B_j(x, D) := \sum_{|\alpha| \leq k_j} b_{j,\alpha}(x) D^\alpha$, $b_{j,\alpha}(x) \in C^{2m-k_j}(\partial\Omega)$ such, that $0 \leq k_1 < k_2 < \dots < k_m \leq 2m-1$, $k_j < 2m\theta - \frac{1}{p}$, $j = \overline{1, m}$. Further $b_j(x, \xi) := \sum_{|\alpha|=k_j} b_{j,\alpha}(x) \xi^\alpha$.

In space $L_p(\Omega)$, $p \in (1, \infty)$ the linear operator

$$A : W_{p, \{B_j\}}^{2m}(\Omega) \longrightarrow L_p(\Omega), \quad Au := - \sum_{|\alpha| \leq 2m} a_\alpha(x) D^\alpha L(x, D)u(x),$$

$$W_{p, \{B_j\}}^{2m}(\Omega) := \left\{ u(x) \in W_p^{2m}(\Omega) : B_j(x, D)u(x)|_{\partial\Omega} = 0, j = \overline{1, m} \right\}$$

is considered. We assume, that the number $\omega_0 \in [0, \pi/2)$ is given and $\forall \omega \in [\omega_0, 2\pi - \omega_0]$ the following conditions are satisfied: **(i)** $\arg a(x, \xi) = 0$, $\forall x \in \overline{\Omega}$, $0 \neq \xi \in \mathbb{R}^n$; **(ii)** for all normal and tangent vectors $\nu_x, \mu_x \neq 0$, ($x \in \partial\Omega$) the polynome $z \rightarrow a(x, \mu_x + z\nu_x) - \lambda$, ($\arg \lambda = \omega$) has equally m roots with the positive imaginary part $z_1, \dots, z_m \in \mathbb{C}$; **(iii)** the polynoms $b_j(x, \mu_x + z\nu_x)$ are linearly independent by module $\prod_{k=1}^m (z - z_k)$.

Let $0 < \theta < 1/2$ and on the subspace $H_{p, \{B_j\}}^{2m\theta}(\Omega) := \{u(x) \in H_p^{2m\theta}(\Omega) : B_j(x, D)u(x)|_{\partial\Omega} = 0; j = \overline{1, m}\}$ in space of Bassel potentials $H_p^{2m\theta}(\Omega)$ the operator $X \in \mathcal{L}(H_{p, \{B_j\}}^{2m\theta}(\Omega); L_p(\Omega))$ is given. We research dependence of the solutions of the mixed non-homogeneous problem

$$\frac{\partial w_s(x, t)}{\partial t} = (A + sX) w_s(x, t) + f(x, t), w_s(x, 0) = g_0(x) \in W_{p, \{B_j\}}^{2m}(\Omega). \quad (1)$$

from the perturbing operators sX . Is proved, if $f(x, t) \in C([0, T]; H_{p, \{B_j\}}^{2m\alpha}(\Omega))$, then $\forall X, s > 0$ there is the unique solution $w_s(x, t) \in$

$C([0, T]; L_p(\Omega)) \cap C^1([0, T]; W_{p, \{B_j\}}^{2m}(\Omega))$ of the problem (1) and this solution satisfies to the condition

$$\lim_{s \rightarrow 0} \|w_s(x, t) - w_0(x, t)\|_{L_p(\Omega)} = 0,$$

where $w_0(x, t)$ is the solution of the appropriate not perturbed problem ($s = 0$). If stronger condition $f(x, t) \in C([0, T]; W_{p, \{B_j\}}^{2m}(\Omega))$ is carried out, then the solution $w_s(x, t)$ of the problem (1), besides satisfies the inequality

$$\sup_{s > 0} \|w_s(x, t)\|_{L_p(\Omega)} \leq K \|w_0(x, t)\|_{L_p(\Omega)}$$

with some constant K .

On the distributions of exponential type

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Dual spaces and convolution algebra structures in dual spaces of linear continuous functionals in the spaces of entire exponential type analytic functions, summable on the real subspaces, is described.

Let us correspond for arbitrary $\nu > 0$ in a space $L_p(R)$, ($1 \leq p < \infty$) a Banach space

$$E_p^\nu(D) \equiv \{f \in L_p(R) \mid \|f\|_\nu \equiv \sum_{k=0}^{\infty} \nu^{-k} \|D^k f\|_{L_p(R)} < \infty\}.$$

It is known that $E_p^\nu(D)$ consists of entire functions of exponential type $< \nu$ that belongs to $L_p(R)$ on real axis [1].

For any vector $\nu = (\nu_1, \dots, \nu_n)$, $\nu_j > 0$ let's define the subspace

$$E_p^\nu(\mathbf{R}^n) = \left\{ \varphi \in L_p(\mathbf{R}^n) \mid \|\varphi\|_{E_p^\nu} = \sup_{k \in \mathbb{Z}_+^n} \frac{\|D^k \varphi\|_{L_p}}{\nu^k} < \infty \right\}.$$

Spaces $E_p^\nu(\mathbf{R}^n)$ are Banach ones and they are invariant respectively to the operators of functional differentiation $D_j^{k_j}$.

From another side let's consider the space \mathcal{M}_p^ν of entire functions $\Phi: \mathbb{R}^n \ni t \mapsto \Phi(t + i\tau)$ with norm

$$\|\Phi\|_{\mathcal{M}_p^\nu} = \sup_{\tau \in \mathbb{R}^n} \exp\left(\sum_{j=1}^n -\nu_j |\tau_j|\right) \left[\int_{\mathbb{R}^n} |\Phi(t + i\tau)|^p dt \right]^{1/p}.$$

Spaces \mathcal{M}_p^ν consist of the functions of exponential type.

Theorem 1. (i) *The mapping $\mathcal{M}_p^\nu \ni \Phi(t + i\tau) \mapsto \varphi(t) = \Phi(t + i0) \in E_p^\nu$ is the isometric of normed spaces.*

(ii) *The embeddings $E_p^\nu \subset L_p(\mathbb{R}^n)$ are isometric.*

Let's consider the union of the spaces with topology of inductive limit $E_p(\mathbb{R}^n) = \lim_{\nu \rightarrow \infty} \text{ind } E_p^\nu$. Adjoint space to the $E_p(\mathbb{R}^n)$ denote by $E'_p(\mathbb{R}^n)$ and endow by weak topology. Functionals $f \in E'_p$ call distributions of exponential type.

Theorem 2. *Let $f, g \in E'_p$ and $\varphi \in E_p$. The space E'_p is a commutative algebra with respect to convolution, that define by relation $(f * g) \star \varphi = f \star (g \star \varphi)$. The mapping $E'_p \ni f \mapsto K_f \in \mathcal{L}(E_p)$, where $K_f \varphi = f$, is an algebraic isomorphism onto the commutant of the group T_{-s} in the algebra $\mathcal{L}(E_p)$.*

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Invariant measures on paratopological groups

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Invariant measures on locally compact groups play an important role in the theory of topological groups and harmonic analysis. In this talk we shall discuss the problem of the existence of an invariant Borel measure on a paratopological group.

Recall that a *paratopological group* is a group G endowed with a topology τ making the group operation continuous. If, in addition, the operation of

taking inverse is continuous, then G is a *topological group*. A Borel measure μ on a paratopological group G is *invariant* if $\mu(xB) = \mu(Bx) = \mu(B)$ for any Borel subset $B \subset G$ and any $x \in G$. A measure μ is *Radon* if for every $\varepsilon > 0$ there is a compact subset $K \subset G$ with $\mu(G \setminus K) < \varepsilon$.

It turns out that the existence of an invariant measure imposes very strict restrictions on the structure of a paratopological group.

Following Guran we called a paratopological group *saturated* if $\text{int}(U^{-1}) \neq \emptyset$ for any $U \subset G$; is *totally bounded* if for any $U \in \tau$ there is finite $A \subset G$ with $AU = G$.

Theorem 1. *Let G be a saturated paratopological group. If G admits a finite Radon invariant measure, then G is a compact topological group.*

Theorem 2. *Let G be a saturated paratopological group. If G admits a σ -finite Radon invariant measure, then G is a locally compact topological group.*

Theorem 3. *Let G be a saturated paratopological group. If G admits a finite r -additive invariant measure, then G is totally bounded.*

Theorem 4. *Let G be a saturated paratopological group. If G admits a σ -finite τ -additive invariant measure, then G is ω -bounded and locally totally bounded.*

Example. The standard Haar measure A on the circle $\mathbb{S} \doteq \{z \in \mathbb{C} \mid |z| = 1\}$ endowed with the Sorgenfrey topology is an example of an invariant Borel measure which is τ -additive but not Radon.

The structure of the direct limits of inverse symmetric semigroups with strictly diagonal embeddings of p -order

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We use standard denotation from [2]. Let IS_n be the inverse symmetric semigroup on the set $\{1, 2, 3, \dots, n\}$. For any $a = \begin{pmatrix} 1 & \cdots & n \\ i_1 & \cdots & i_n \end{pmatrix} \in IS_{p^n}$ we define, similarly [1], an embedding $d^p : IS_{p^n} \rightarrow IS_{p^{n+1}}$ by

$$d^p(a) = \left(\begin{array}{ccc|ccc} 1 & \cdots & p^n & \cdots & (p-1)p^n + 1 & \cdots & (p-1)p^n + p^n \\ i_1 & \cdots & i_{p^n} & \cdots & (p-1)p^n + i_1 & \cdots & (p-1)p^n + i_{p^n} \end{array} \right),$$

where $rp^n + i_k = \emptyset$ for $i_k = \emptyset$, ($k = 1, 2, \dots, p^n$; $r = 1, 2, 3, \dots, p-1$).

The limit semigroup $\lim(IS_{p^k}, d_p)$ of the direct spectrum $(IS_{p^k}, d_p)_{k \in \mathbb{N}}$ is called *inverse symmetric diagonal semigroup of p -order* and is denoted by IS_{p^∞} . There exists a natural embedding of the semigroup IS_{p^∞} into the inverse symmetric semigroup $IS(N)$ over the set of positive integers.

The principal ideals and Green's relations of the inductive limits of the finite semigroups of partial transformations with diagonal embeddings of p -order are described in [3]. Here we give the same characteristics for IS_{p^∞} .

Theorem 1. 1) The principal left ideal $L(a)$ generated by the element a of the semigroup IS_{p^∞} consists of such elements b of IS_{p^∞} that $\text{ran}(b) \subseteq \text{ran}(a)$;

2) The principal right ideal $R(a)$ generated by the element a of the semigroup IS_{p^∞} consists of such elements b of IS_{p^∞} that $\text{dom}(b) \subseteq \text{dom}(a)$;

3) The principle two-sides ideal $I(a)$ generated by the element a of the semigroup IS_{p^∞} consists of such elements b of IS_{p^∞} that $\text{def}(b) \geq \text{def}(a)$.

Green's relations R, L, H, D, J of this semigroup are described in the next theorem.

Theorem 2. For elements a, b of the semigroup IS_{p^∞} it is true that:

1) aRb if and only if $\text{dom}(a) = \text{dom}(b)$;

2) aLb if and only if $\text{ran}(a) = \text{ran}(b)$;

3) aHb if and only if $\text{dom}(a) = \text{dom}(b)$ and $\text{ran}(a) = \text{ran}(b)$;

4) aDb if and only if $r(a) = r(b)$;

5) $D = I$.

Elements a, b of the semigroup IS_{p^∞} are called *conjugate* if there exists a permutation $g \in IS_{p^\infty}$ such that $b = g^{-1}ag$.

Theorem 3. Two partial permutations $a, b \in IS_{p^\infty}$ are conjugated if and only if the restriction of these permutations on the some finite (a, b) -invariant subset $\{1, 2, 3, \dots, l\}$ of the N have the same cycle and chain type.

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Spectral properties of composition operators in a Hilbert space

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Space of analytic functions of infinitely many variables are standard object in functional analysis. The main results of the theory of these spaces are studied in monograph [1]. Among linear operators there are composition operators which act on the spaces of analytic functions. Our purpose is to show spectral properties of composition operators in a Hilbert space of analytic functions on a separable Hilbert space.

Let E be complex Hilbert space with the inner product $(\cdot | \cdot)$ and orthonormal basis (e_k) . Denote by E^∞ the symmetric Fock space which is the ℓ_2 – sum of the symmetric tensor products $\otimes_s^n E$ of E for $n = 0, 1, \dots, \infty$.

In [2] there was shown that for every element $w \in E^\infty$ there exists an analytic function f so that

$$f(x) = \langle \eta(x) | w \rangle, \quad (1)$$

where $\eta(x) = 1 + x + \dots + \underbrace{x \otimes \dots \otimes x}_n + \dots$, $x \in E$ and $\langle \cdot | \cdot \rangle$ is inner product in E^∞ .

Let $\mathcal{H}^2(E)$ denote the space of analytic functions defined by formula (1). Note that $\mathcal{H}^2(E) = (E^\infty)'$. Let $F: E \rightarrow E$ be a linear operator and operator $T_F: \mathcal{H}^2(E) \rightarrow \mathcal{H}^2(E)$ so that $T_F(f)(x) = f(F(x))$.

Theorem 1. *Let be $F: B \rightarrow B$, where B is the unit ball. Then T_F is a selfadjoint operator if and only if F is a linear selfadjoint operator.*

Theorem 2. *If $\lambda_1, \dots, \lambda_n, \dots$ is sequence of eigenvalues of compact selfadjoint operator F then $\lambda_{k_1} \dots \lambda_{k_n}$ are eigenvalues of operator T_F .*

Theorem 3. *If T_F is selfadjoint operator on $\mathcal{H}^2(B)$ then there exists $\underbrace{\mathcal{E}_t \otimes \dots \otimes \mathcal{E}_t}_n$ decomposition of unit in E^n so that*

$$T_F = \sum_{n=1}^{\infty} \int_{\sigma(F)} \dots \int_{\sigma(F)} \lambda_1 \dots \lambda_n d\mathcal{E}(\lambda_1) \otimes \dots \otimes d\mathcal{E}(\lambda_n).$$

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Absolutely H -closed topological semigroups and Brandt λ -extensions

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We follow the terminology of [1, 2]. All spaces are Hausdorff.

Let S be a semigroup and let I_λ be a set of cardinality $\lambda \geq 2$. On the set $B_\lambda(S) = I_\lambda \times S^1 \times I_\lambda \cup \{0\}$ we define the semigroup operation ' \cdot ' as follows

$$(\alpha, a, \beta) \cdot (\gamma, b, \delta) = \begin{cases} (\alpha, ab, \delta) & \text{if } \beta = \gamma \\ 0 & \text{if } \beta \neq \gamma \end{cases}$$

and $(\alpha, a, \beta) \cdot 0 = 0 \cdot (\alpha, a, \beta) = 0 \cdot 0 = 0$ for $\alpha, \beta, \gamma, \delta \in I_\lambda$, $a, b \in S^1$. The semigroup $B_\lambda(S)$ is called a *Brandt λ -extension of the semigroup S* [4].

Let \mathcal{S} be a class of topological semigroups.

Definition 1 [5]. Let λ be a cardinal ≥ 2 , and $(S, \tau) \in \mathcal{S}$. Let τ_B be a topology on $B_\lambda(S)$ such that $(B_\lambda(S), \tau_B) \in \mathcal{S}$ and $\tau_B|_{(\alpha, S^1, \alpha)} = \tau$ for some $\alpha \in I_\lambda$. Then $(B_\lambda(S), \tau_B)$ is called a *topological Brandt λ -extension of (S, τ) in \mathcal{S}* . If \mathcal{S} coincides with the class of all topological semigroups, then $(B_\lambda(S), \tau_B)$ is called a *topological Brandt λ -extension of (S, τ)* .

Recall [3, 4], a semigroup $S \in \mathcal{S}$ is called *H -closed in \mathcal{S}* , if S is a closed subsemigroup of any topological semigroup $T \in \mathcal{S}$ which contains S as a subsemigroup. If \mathcal{S} coincides with the class of all topological semigroups, then the semigroup S is called *H -closed*.

Definition 2 [3, 6]. A topological semigroup $S \in \mathcal{S}$ is called *absolutely H -closed in the class \mathcal{S}* , if any continuous homomorphic image of S into $T \in \mathcal{S}$ is H -closed in \mathcal{S} . If \mathcal{S} coincides with the class of all topological semigroups, then the semigroup S is called *absolutely H -closed*.

For each $\alpha, \beta \in I_\lambda$ we define $V_\alpha = B_\lambda(S) \setminus \{(\alpha, s, \gamma) \mid s \in S^1, \gamma \in I_\lambda\}$ and $H_\beta = B_\lambda(S) \setminus \{(\gamma, s, \beta) \mid s \in S^1, \gamma \in I_\lambda\}$. Put $U^{\alpha_1, \dots, \alpha_n} = \bigcap_{i=1}^n V_{\alpha_i}$, $U_{\beta_1, \dots, \beta_m} = \bigcap_{i=1}^m H_{\beta_i}$ and $U_{\beta_1, \dots, \beta_m}^{\alpha_1, \dots, \alpha_n} = U^{\alpha_1, \dots, \alpha_n} \cap U_{\beta_1, \dots, \beta_m}$, where $\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \in I_\lambda$, $n, m \in \mathbb{N}$. Let (S, τ) be a topological semi-group and let \mathcal{B} be a base of the topology τ . Further we define the following families

$$\begin{aligned} \mathcal{B}_{mv} &= \{U^{\alpha_1, \dots, \alpha_n} \mid \alpha_1, \dots, \alpha_n \in I_\lambda, n \in \mathbb{N}\} \cup \{(\alpha, V, \beta) \mid V \in \mathcal{B}, \alpha, \beta \in I_\lambda\}, \\ \mathcal{B}_{mh} &= \{U_{\beta_1, \dots, \beta_m} \mid \beta_1, \dots, \beta_m \in I_\lambda, m \in \mathbb{N}\} \cup \{(\alpha, V, \beta) \mid V \in \mathcal{B}, \alpha, \beta \in I_\lambda\}, \\ \mathcal{B}_{mi} &= \{U_{\beta_1, \dots, \beta_m}^{\alpha_1, \dots, \alpha_n} \mid \alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \in I_\lambda, n, m \in \mathbb{N}\} \cup \\ &\quad \cup \{(\alpha, V, \beta) \mid V \in \mathcal{B}, \alpha, \beta \in I_\lambda\}. \end{aligned}$$

Obviously, the conditions (BP1)—(BP3) [2] hold for families \mathcal{B}_{mv} , \mathcal{B}_{mh} and \mathcal{B}_{mi} , and hence \mathcal{B}_{mv} , \mathcal{B}_{mh} and \mathcal{B}_{mi} are bases on $B_\lambda(S)$ of topologies $\tau_{mv}(S)$, $\tau_{mh}(S)$ and $\tau_{mi}(S)$, respectively.

In [4] O. Gutik and K. Pavlyk proved that if (S, τ) is an H -closed topological semigroup then $(B_\lambda(S), \tau_{mv}(S))$, $(B_\lambda(S), \tau_{mh}(S))$ and $(B_\lambda(S), \tau_{mi}(S))$ are H -closed topological semigroups. Also in [5] they showed that for topological inverse semigroup S the following conditions are equivalent: (i) S is an absolutely H -closed topological semigroup in the class of topological inverse semigroups; (ii) there exists a cardinal $\lambda \geq 2$ such that any topological Brandt λ -extension $B_\lambda(S)$ is an absolutely H -closed topological semigroup in the class of topological inverse semigroups; (iii) for any cardinal $\lambda \geq 2$ every topological Brandt λ -extension $B_\lambda(S)$ is an absolutely H -closed topological semigroup in the class of topological inverse semigroups.

Theorem. *Let λ be an infinite cardinal and let (S, τ) be an absolutely H -closed topological semigroup. Then $(B_\lambda(S), \tau_{mv}(S))$, $(B_\lambda(S), \tau_{mh}(S))$ and $(B_\lambda(S), \tau_{mi}(S))$ are absolutely H -closed topological semigroups.*

Corollary 1. *Let λ be an infinite cardinal and let (S, τ) be an absolutely H -closed topological inverse semigroup. Then $(B_\lambda(S), \tau_{mi}(S))$ is absolutely H -closed inverse topological semigroup.*

Corollary 2. *Let λ be an infinite cardinal and let (S, τ) be a compact topological semigroup. Then $(B_\lambda(S), \tau_{mv}(S))$, $(B_\lambda(S), \tau_{mh}(S))$ and $(B_\lambda(S), \tau_{mi}(S))$ are absolutely H -closed topological semigroups.*

Corollary 3. *Let λ be an infinite cardinal and let (S, τ) be a compact topological inverse semigroup. Then $(B_\lambda(S), \tau_{mi}(S))$ is absolutely H -closed topological inverse semigroup.*

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Verbal subgroups of the group of 1-triangular matrices

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We use such denotes: G_q is a group of 1-triangular matrixes over field F_q with q elements, G is a group of 1-triangular matrixes over any field F , $\gamma_k(G)$ is an element of low central series of G , $V(G)$ is a verbal subgroup of group G , generated by set of words V , $v(G)$ is a verbal subgroup of group G , generated by some word v .

Theorem.

1. Subgroup $v(G)$ is equal to G if and only if the sum of degree indexes of some letter of v is equal to $t \neq 0$ as element of the field F . In this case width of $v(G)$ is equal to 1, and for any matrix $A \in G$ it is possible to find explicit form of such matrix $B \in G$, that $B^t = A$, and prove that such matrix B is unique.
2. For any set V there exists such k ($1 \leq k \leq n$), that $V(G_q) = \gamma_k(G_q)$.
3. If $v = a^s$, then $v(G_2) = \gamma_{2^k}(G_2)$, where $s = 2^k \cdot r$, $r \equiv 1 \pmod{2}$. In this case if $2^k + 3 \leq n$ then width of $a^s(G_2)$ is not equal to 1.

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Maximal ballean

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A *ball structure* is a triplet $\mathbb{B} = (X, P, B)$, where X, P are non-empty sets and, for any $x \in X$, $\alpha \in P$, $B(x, \alpha)$ is a subset of X which is called a ball of radius α around x . It is supposed that $x \in B(x, \alpha)$ for all $x \in X$, $\alpha \in P$. Given any $x \in X$, $\alpha \in P$ we put $B^*(x, \alpha) = \{y \in X : x \in B(y, \alpha)\}$.

A ball structure \mathbb{B} is called a *ballean* if

- for any $\alpha, \beta \in P$, there exist $\alpha', \beta' \in P$ such that, for every $x \in X$,

$$B(x, \alpha) \subseteq B^*(x, \alpha'), \quad B^*(x, \beta) \subseteq B(x, \beta');$$

- for any $\alpha, \beta \in P$, there exist $\gamma \in P$ such that, for every $x \in X$,

$$B(B(x, \alpha), \beta) \subseteq B(x, \gamma).$$

A subset $A \subseteq X$ is called *bounded* if $A \subseteq B(x, \alpha)$ for some $x \in X$, $\alpha \in P$. We say that \mathbb{B} is bounded if its support X is bounded. A ballean \mathbb{B} is called *connected* if, for any $x, y \in X$, there exists $\alpha \in P$ such that $y \in B(x, \alpha)$. A ballean \mathbb{B} is called *proper* if \mathbb{B} is unbounded and connected.

Let $\mathbb{B} = (X_1, P_1, B_1)$, $\mathbb{B}' = (X', P', B')$ be ballean with common support X . We write $\mathbb{B} \preceq \mathbb{B}'$ if, for every $\alpha \in P$, there exists $\alpha' \in P'$ such that $B(x, \alpha) \subseteq B(x, \alpha')$, for every $x \in X$. If $\mathbb{B}' \preceq \mathbb{B}$ and $\mathbb{B} \preceq \mathbb{B}'$, we write $\mathbb{B} = \mathbb{B}'$.

A proper ballean \mathbb{B} is called *maximal* if for every proper ballean \mathbb{B}' , $\mathbb{B} \preceq \mathbb{B}'$ implies $\mathbb{B} = \mathbb{B}'$. We denote by \mathcal{M}_0 the class of all maximal ballean and introduce three classes of proper ballean close to \mathcal{M}_0 by the explicit description of their members:

\mathcal{M}_{-1} : there exists only one ultrafilter p on X , going to infinity in \mathbb{B} (i.e. $X \setminus A \in p$ for every bounded subset A);

\mathcal{M}_1 : every unbounded subset $A \subseteq X$ is large (i.e. there exists $\alpha \in P$ such that $B(A, \alpha) = X$, where $B(A, \alpha) = \bigcup_{a \in A} B(a, \alpha)$);

\mathcal{M}_2 : the corona $\nu(\mathbb{B})$ is a singleton (for definition of corona see [1, 2]).

We prove that $M_{-1} \subset M_0 \subset M_1 \subset M_2$. To show that these inclusions are strict we propose some general trick for strengthening of ballean.

A proper ballean $\mathbb{B} = (X, P, B)$ is called *irresolvable* if X can not be partitioned into two large subset. A ballean \mathbb{B} is irresolvable if and only if \mathbb{B} is pseudodiscrete (i.e. for every $\alpha \in P$, there exists a bounded subset $A \subseteq X$ such that $B(x, \alpha) = \{x\}$ for every $x \in X \setminus A$). The classes of maximal and irresolvable ballians have the natural counterparts in topology, but in contrast to the topological situation, these classes are not incident.

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A method for constructing examples of M -equivalent mappings

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Denote by $F(X)$ the free topological group [1] over a Tychonoff space X . We call two mappings $f: X_1 \rightarrow Y_1$ and $g: X_2 \rightarrow Y_2$ M -equivalent (written $f \stackrel{M}{\sim} g$) [1] if there exist topological isomorphisms $i: F(X_1) \rightarrow F(X_2)$ and $j: F(Y_1) \rightarrow F(Y_2)$ such that $j \circ f^* = g^* \circ i$ where $f^*: F(X_1) \rightarrow F(Y_1)$ and $g^*: F(X_2) \rightarrow F(Y_2)$ are homomorphisms extending f and g .

Theorem. *Let X be a Tychonoff space and r_1 and r_2 its retractions onto the same retract K . Then $r_1 \stackrel{M}{\sim} r_2$.*

Recall that a map $f: X \rightarrow Y$ is *finite-to-one* (compact, monotone) if any $f^{-1}(y)$ is finite (compact, connected). A closed compact map is called *perfect*. A map f is called a *local homeomorphism* if for any $x \in X$ there exists its neighbourhood $U(x)$ such that $f|_{U(x)}$ is a homeomorphism of $U(x)$ onto an open subset in Y . We say that $\dim(f) \leq n$ if $\dim(f^{-1}(y)) \leq n$ for any y .

The theorem allows us to show that the following properties are not preserved by the relation of M -equivalence: compactness, perfectness, monotonicity, dimension, finite-to-one property, being a local homeomorphism.

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On algebraic structures on some k_ω -spaces

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We consider infinite-dimensional manifolds modeled on some nonmetrizable absolute extensors (see [1] for results on model spaces). A general problem of existence of algebraic structures compatible with the topology of these spaces is investigated. A special attention is paid to the (semi-)group structures.

We also extend to the case of k_ω -spaces the problem of topological homogeneity of spaces of the form $F(\varinjlim X_i)$, where X_i are uncountable powers of absolute extensors and F is a normal functor in the category of k_ω -spaces [2].

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About forming d -fields of differential invariants ruled surface for action of group $G = O(3, R)$

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Let $G = O(3, R)$ be the orthogonal group of the third order, above R . Ruled surface in $V = R^3$ we shall name display $z : R \times R \rightarrow V$, set by the formula: $z(t, u) = x(t) + uy(t)$, where $x(t) = (x_1(t), x_2(t), x_3(t))$, $y(t) = (y_1(t), y_2(t), y_3(t))$, x_i, y_j , $(i, j = 1, 2, 3)$ - indefinitely differentiated functions on R . A polynomial from $z(t, u)$ and final number of derivatives from $z(t, u)$ on t and on u we shall name a differential polynomial (d -polynomial) from a ruled surface $z(t, u)$. The set of differential polynomials from a ruled surface $z(t, u)$ forms differential ring $C\{V\}$. The relation of d -polynomials from $z(t, u)$ we shall name differential rational function from a ruled surface $z(t, u)$. The set of differential rational functions from a ruled surface $z(t, u)$ forms a differential field (d -field). We shall designate it through $C < V >$. We shall define action of group G in $C < V >$ under the formula: $(T_g f)\langle z(t, u) \rangle = f\langle z(t, u)g \rangle$, where $g \in G$. Let $(C\langle V \rangle)^G$ -subfield in $C\langle V \rangle$, consisting from G -invariant differential rational functions. For classical groups the problem about a finding of final number forming for fields similar to a field $(C\langle V \rangle)^G$ is put [1].

Theorem 1. *Differential forming d -fields $(C\langle V \rangle)^G$ the following d -polynomials are: $(x + uy, x + uy)$, $(\dot{x} + u\dot{y}, \dot{x} + u\dot{y})$, $(\ddot{x} + u\ddot{y}, \ddot{x} + u\ddot{y})$, $(\dot{x} + u\dot{y}, y)$.*

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