

Immiserizing Growth: A Geometrical Note

The effect of economic expansion on international trade has been receiving increasing attention from economic theorists since the publication of Professor Hicks' stimulating analysis of the "dollar problem".¹ It has, however, been insufficiently realised that, under certain circumstances, economic expansion may harm the growing country itself.² Economic expansion increases *output* which, however, might lead to a sufficient deterioration in the terms of trade to offset the beneficial effect of expansion and reduce the *real income* of the growing country. It is the purpose of this note to formulate the conditions

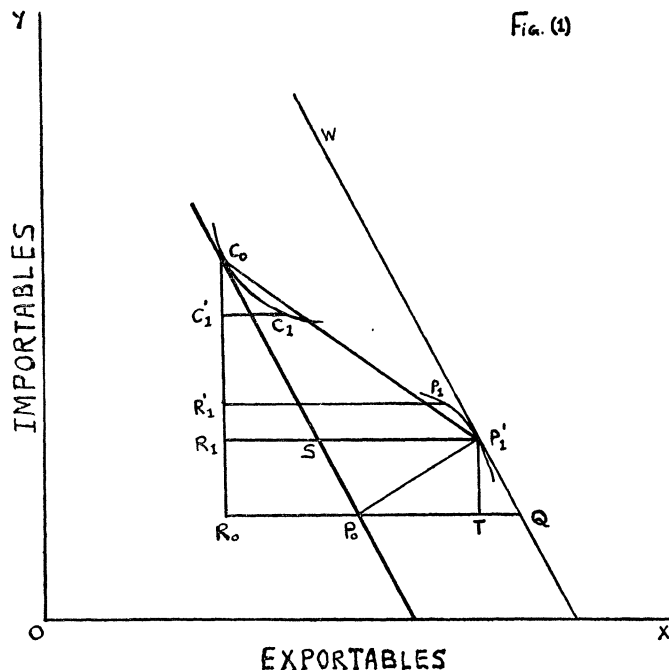


FIGURE 1

¹ J. R. Hicks, "An Inaugural Lecture," *Oxford Economic Papers*, N.S. Vol. 5, No. 2 (June, 1953). The following are of interest: H. G. Johnson, "Economic Expansion and International Trade," *The Manchester School of Economic and Social Studies*, May, 1955; E. J. Mishan, "The Long-Run Dollar Problem: A Comment," *Oxford Economic Papers*, N.S. Vol. 7, No. 2 (June, 1955); and W. M. Corden, "Economic Expansion and International Trade: A Geometric Approach," *Oxford Economic Papers*, N.S. Vol. 8, No. 2 (June, 1956).

² Exception must be made, however, in the case of Professor Johnson, "Equilibrium Growth in An Expanding Economy," *The Canadian Journal of Economics and Political Science*, Vol. XIX, No. 4, (Nov., 1953), p. 495; and also his *Manchester School*, May 1955, article. It should also be mentioned that Prof. Johnson has independently worked out mathematically, in an unpublished note, a criterion for immiserizing growth which confirms the results derived geometrically in this note.

under which immiserizing growth will occur. Section I sets out the analysis geometrically and arrives at the criterion for immiserizing growth. Section II discusses some of the implications of this criterion.

I

In the ensuing analysis we assume the traditional two-country, two commodity "real" model where full-employment always obtains. We also assume, to simplify the analysis, that growth is confined to a single country so that the other country (i.e., the rest-of-the-world) is not experiencing any growth in *output*; this assumption enables us to assume the offer curve of the rest-of-the-world as "given" during the course of our analysis. Finally, we simplify the problem by beginning with an investigation of the conditions under which growth would leave the country just as well off as before, and then determining whether the equilibrium actually realised would involve still less favourable terms of trade; this approach has the convenience of avoiding the need for an explicit analysis of the income effect of growth.

Consider now Fig. (1) which represents the growing economy. C_0 is the pre-expansion consumption point, P_0 the pre-expansion production point, P_0C_0 the pre-expansion terms of trade or price-line, C_0R_0 the imports of Y into the country and R_0P_0 the exports of X from the country. The production-possibility curve tangential to P_0C_0 has not been drawn in to avoid cluttering up the diagram; the indifference curve through C_0 is tangential to P_0C_0 at C_0 and has been drawn partially. Consider now growth which pushes the production-possibility curve outwards and which, at *constant terms of trade*, would bring production from P_0 to P'_1 . Now assume that the terms of trade are changed just enough to offset indifference and the *new* production-possibility curve. We later assume, legitimately for infinitesimal changes, that C_1P_1 coincides with $C_0P'_1$.

The combined effect of the expansion and the compensating adjustment of the terms of trade is to reduce the demand for imports from C_0R_0 to C'_1R_1 . This reduction can be analysed into the sum of three effects:

(1) *The increase in production of importables due to the economic expansion:*

This increase (R_0R_1 in the diagram) may be analysed as follows. Let p_0 and p_1 be the original and the *zero-gain* prices respectively, measured as the number of units of exportables required to buy a unit of importables. Then the change in total output, valued at *initial* prices, is:

$$P_0T + TQ = P_0Q = SP'_1$$

And

$$SP'_1 = \frac{P'_1R_1 - R_1S}{C_0R_1} \cdot C_0R_1 = (p_1 - p_0) \cdot C_0R_1$$

The change in the production of importables is:

$$R_0R_1 = P'_1T = \frac{\delta Y}{\delta K} \cdot P_0Q = \frac{\delta Y}{\delta K} \cdot SP'_1$$

where K is defined to be the country's productive capacity which is assumed to be kept fully employed and is measured by the value in terms of exportables of the output the

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In “Immiserizing Growth : A Geometrical Note”, by J. Bhagwati, this Review, June 1958

- (i) Read “the gain from growth ; the relevant price-line being C_1F_1 which is tangential to the old” after line 8, second paragraph, Section 1, p. 202.

country would produce at the initial terms of trade and Y is the domestic output of importables. Then,

$$R_0R_1 = C_0R_1 \cdot \frac{\delta Y}{\delta K} \cdot (p_1 - p_0)$$

Since we have assumed the changes to be infinitesimal, it follows that we can assume $C_0R_1 = C_0R_0$, the initial volume of imports, so that

$$R_0R_1 = M \cdot \frac{\delta Y}{\delta K} \cdot dp \quad (S_m \equiv M) \quad (1)$$

where M is the quantity of imports.

This shows the change in the production of importables due to the economic expansion itself. The expression is normally positive, indicating that the output of importables increases, consequent on economic expansion, at constant terms of trade. It should be noted here, however, that, as argued in Section II, the output of importables may actually contract due to the expansion.

(2) *The decrease in consumption of importables due to the price-change :*

The price-change (from p_0 to p_1) shifts consumption *along* the indifference curve to C_1 . The consumption of importables is then reduced by:

$$C_0C_1 = - \frac{\delta C}{\delta p} \cdot dp \quad (2)$$

where C is the total demand for importables.

(3) *The increase in production of importables due to the price-change:*

The price-change shifts production *along* the production-possibility curve to P_1 . The production of importables is then increased by:

$$R_1R'_1 = \frac{\delta Y}{\delta p} \cdot dp \quad (3)$$

The total decrease in the domestic demand for *imports*¹ is the sum of the three effects (1), (2) and (3) :

$$\left(M \cdot \frac{\delta Y}{\delta K} + \frac{\delta Y}{\delta p} - \frac{\delta C}{\delta p} \right) \cdot dp \quad (4)$$

This expression measures the decrease in demand for imports when the effect of growth on real income is exactly offset by an adverse movement of the terms of trade. In the abnormal case where output of importables *falls* as a result of growth, the expression may be negative, indicating an *increase* in the demand for imports.

Whether the country will actually be made worse off or not depends on what would happen to the quantity of imports supplied if the terms of trade were adjusted as assumed. The change in imports supplied as a result of such a price change is :

$$\frac{\delta S_m}{\delta p} \cdot dp \quad (5)$$

¹ As distinguished from *importables*.

The sum of (4) and (5) constitutes the excess supply of imports at the zero-gain terms of trade : if it is positive, the terms of trade will not move against the growing country enough to deprive it of all gain from growth ; but if it is negative, the price of imports will have to rise still further to preserve equilibrium, and the growing country will actually be made worse off by growth.

The economic meaning of this criterion for immiserizing growth will be considered in the next section ; for this purpose a neater formulation of the criterion is desirable, and this can be derived by subjecting it to some algebraic manipulation.

Multiplying (4) and (5) by $\frac{p}{M \cdot dp}$, we get our criterion for immiserizing growth as :

$$\left(\frac{C}{M} \cdot \varepsilon + \frac{Y}{M} \cdot \sigma + y + r_m \right) < 0 \quad (6)$$

which may be written as:

$$\left(\frac{C}{M} \cdot \varepsilon + \frac{Y}{M} \cdot \sigma + y \right) < -r_m \quad (7)$$

where

$$\varepsilon = -\frac{p}{C} \cdot \frac{\delta C}{\delta p}, \quad r_m = \frac{p}{M} \cdot \frac{\delta S_m}{\delta p} \quad (S_m \equiv M)$$

$$\sigma = \frac{p}{Y} \cdot \frac{\delta Y}{\delta p} \quad \text{and} \quad y = p \cdot \frac{\delta Y}{\delta K}$$

This criterion is also expressible in the alternative equivalent form :

$$\left(\frac{C}{M} \cdot \varepsilon + \frac{Y}{M} \cdot \sigma + y \right) < 1 - \eta_x \quad (8)$$

where $\eta_x = \frac{p}{X^\circ} \cdot \frac{\delta X^\circ}{\delta p}$ and X° is the quantity of exports. This follows from the fact that η_x and r_m are the *total* elasticities of the rest-of-the-world's offer curve ; η_x being the elasticity of the rest-of-the-world's demand for imports (into the rest-of-the-world) in response to an infinitesimal change in the terms of trade and r_m being the elasticity of the rest-of-the-world's supply of (its) exports (to the growing country) in response to an infinitesimal shift in the terms of trade. It is a well-known proposition in the theory of international trade that $\eta_x - r_m = 1$; hence, $1 - \eta_x = -r_m$.

II

What are the implications of the criterion that we have derived in Section I ? It will be remembered that $\sigma = \frac{p}{Y} \cdot \frac{\delta Y}{\delta p}$ and is thus necessarily positive and $\varepsilon = -\frac{p}{C} \cdot \frac{\delta C}{\delta p}$ which again, being the constant-utility or expenditure-compensated demand-elasticity

with respect to a change in the price of importables, is necessarily positive.¹ We can see from (6), (7) or (8) that the *possibility* of immiserizing growth is increased if :

- (i) $\frac{Y}{M}$, the ratio of domestic production to import of importables is small. Since $\frac{C}{M} = 1 + \frac{Y}{M}$, it follows that $\frac{C}{M}$ will also be small when $\frac{Y}{M}$ is small ;
- (ii) ϵ , the constant-utility demand-elasticity for importables with respect to a change in the price of importables, is small ; this would depend on the substitution effect against importables being negligible when the price of importables rises ; and
- (iii) σ , the elasticity in supply of importables when production shifts along the production-possibility curve in response to a change in the price of importables, is small.

These are, neither singly nor in combination, sufficient conditions for immiserizing growth. In fact, the *possibility* of immiserizing growth arises only when, with these conditions favourably fulfilled, either or both of the following crucial conditions are fulfilled :

(a) the offer of the rest-of-the-world is inelastic, (i.e., r_m is negative, which may be for the *extreme*, and by no means necessary, reason that the growing country's exports are Giffen goods abroad) ; and

(b) growth actually reduces the domestic production of importables at constant relative commodity prices (i.e., y is negative).

Stringent as the latter condition may appear at first sight, recent analyses have shown that it is feasible under relatively simple assumptions. Thus the Rybczynski proposition states that under a two-commodity, two-factor model where, say, labour and land being the factors, one good is labour-intensive and the other land-intensive, if labour (land) increases in supply, then the output of the land-intensive (labour-intensive) industry must actually contract if the relative commodity prices are maintained constant.² Professor Johnson has recently advanced the proposition that under neutral technical progress in one industry, the technology of the other and the total factor endowment remaining unchanged, the output of the other industry must actually fall under constant relative commodity prices.³ It may be of interest to note that under biased progress as well it is possible to establish conditions under which the output of the non-innovating industry will contract.⁴

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¹ This argument obviously rests on the assumption of "well-behaved" (convex) indifference curves and (concave) transformation curves, concavity being defined with reference to the origin and *not* in the strict mathematical sense.

² Rybczynski, "Factor Endowments and Relative Commodity Prices," *Economica*, Nov., 1955. Linear homogeneity of the production functions and diminishing returns are *sufficient* conditions for the proposition to hold. The strong Samuelson notion of factor-intensity is not necessary.

³ Johnson, *Manchester School*, *op. cit.* Diminishing returns are *sufficient* for this proposition to hold. The proposition can be readily extended to more than two goods and factors.

⁴ The conditions under which this result will obtain can be established for specified production functions.

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